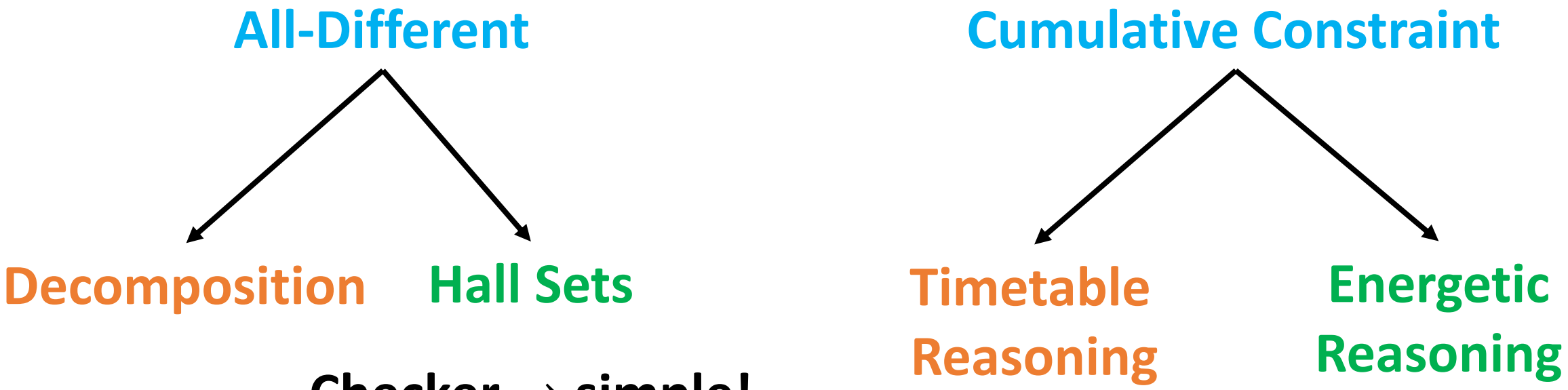
TU Delft

Conflict Analysis

Modern Constraint Programming (ESSAI'26)

Emir Demirović

Last time…



Checker → simple!

Propagation → Flow

Propagation based on Hall sets!

$$x_1 \in \{1, 2\}$$

$$x_2 \in \{1, 2, \cancel{3}\}$$

$$x_3 \in \{3, 4\}$$

$$x_4 \in \{3, 4, 5\}$$

$$x_5 \in \{4, 5\}$$

$$\langle x_3 \geq 3 \rangle \wedge \langle x_3 \leq 5 \rangle \wedge$$
$$\langle x_4 \geq 3 \rangle \wedge \langle x_4 \leq 5 \rangle \wedge$$
$$\langle x_5 \geq 3 \rangle \wedge \langle x_3 \leq 5 \rangle$$
$$\Rightarrow$$
$$\langle x_2 \neq 3 \rangle$$

Checker for All-Different

1. Reconstruct domains based on the explanation

2. Check condition

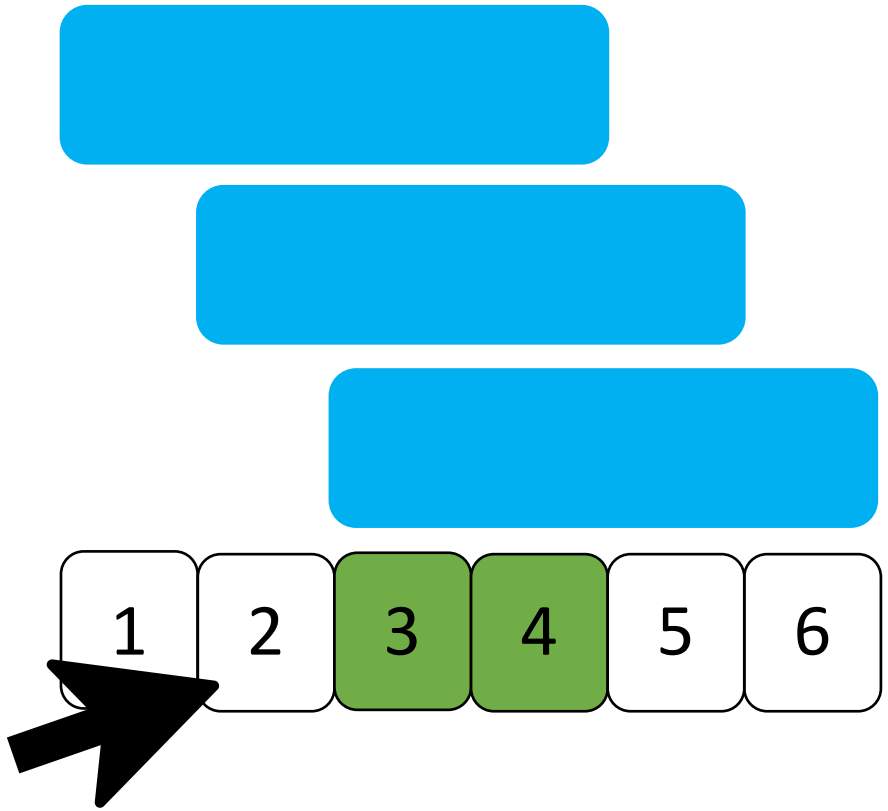
$$H = \{x_i\}$$

$$\bigcup D(x_i) < |H| \, ?$$

Cumulative Propagator

$$s = \{1, 2, 3\}$$

$$D = 4$$



Compulsory resource consumption

In this lecture…

Conflict Analysis

Derive new constraints
by combining existing constraints

Backtrack more than one decision level

Verify learned constraints

Three versions of conflict analysis

(Extended) Nogood Propagation

Hypercube linears

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$ @3

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle y \geq 2 \rangle$ @4

$\langle x_5 \leq 2 \rangle$ @5

$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$c_5 \rightarrow \langle x_8 \geq 1 \rangle$

$c_6 \rightarrow \bot$

Constraints

$c_1 : 3x_2 + x_3 + x_9 \geq 10$

$c_2 : -x_3 + 5x_4 \geq 4$

$c_3 : x_1 + x_5 + 2x_6 \geq 8$

$c_4 : x_7 - x_6 \geq 0$

$c_5 : 2x_8 - x_6 + x_9 \geq 0$

$c_6 : -x_3 - x_4 - x_7 - x_8 \geq -4$

$x_i, y \in \{-10, 10\}$

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$ @3

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle y \geq 2 \rangle$ @4

$\langle x_5 \leq 2 \rangle$ @5

$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$c_5 \rightarrow \langle x_8 \geq 1 \rangle$

$c_6 \rightarrow \bot$

info about search

Constraints

$$c_1 : 3x_2 + x_3 + x_9 \geq 10$$

$$c_2 : -x_3 + 5x_4 \geq 4$$

$$c_3 : x_1 + x_5 + 2x_6 \geq 8$$

$$c_4 : x_7 - x_6 \geq 0$$

$$c_5 : 2x_8 - x_6 + x_9 \geq 0$$

$$c_6 : -x_3 - x_4 - x_7 - x_8 \geq -4$$

$$x_i, y \in \{-10, 10\}$$

Trail

$\langle x_9 \leq 0 \rangle$	@1
$\langle x_1 \leq 2 \rangle$	@2
$\langle x_2 \leq 3 \rangle$	@3
$c_1 \rightarrow \langle x_3 \geq 1 \rangle$	
$c_2 \rightarrow \langle x_4 \geq 1 \rangle$	
$\langle y \geq 2 \rangle$	@4
$\langle x_5 \leq 2 \rangle$	@5
$c_3 \rightarrow \langle x_6 \geq 2 \rangle$	
$c_4 \rightarrow \langle x_7 \geq 2 \rangle$	
$c_5 \rightarrow \langle x_8 \geq 1 \rangle$	
$c_6 \rightarrow \bot$	

Decided $x_9 \leq 0$

Decision level 1

Constraints

$$c_1 : 3x_2 + x_3 + x_9 \geq 10$$

$$c_2 : -x_3 + 5x_4 \geq 4$$

$$c_3 : x_1 + x_5 + 2x_6 \geq 8$$

$$c_4 : x_7 - x_6 \geq 0$$

$$c_5 : 2x_8 - x_6 + x_9 \geq 0$$

$$c_6 : -x_3 - x_4 - x_7 - x_8 \geq -4$$

$$x_i, y \in \{-10, 10\}$$

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$ @3

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle y \geq 2 \rangle$ @4

$\langle x_5 \leq 2 \rangle$ @5

$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$c_5 \rightarrow \langle x_8 \geq 1 \rangle$

$c_6 \rightarrow \bot$

Decided $x_2 \leq 3$

Decision level 3

Constraints

$c_1 : 3x_2 + x_3 + x_9 \geq 10$

$c_2 : -x_3 + 5x_4 \geq 4$

$c_3 : x_1 + x_5 + 2x_6 \geq 8$

$c_4 : x_7 - x_6 \geq 0$

$c_5 : 2x_8 - x_6 + x_9 \geq 0$

$c_6 : -x_3 - x_4 - x_7 - x_8 \geq -4$

$x_i, y \in \{-10, 10\}$

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$ @3

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

Constraint c_1

Propagated $x_3 \geq 1$

Decision level 3

$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$c_5 \rightarrow \langle x_8 \geq 1 \rangle$

$c_6 \rightarrow \bot$

Constraints

$c_1 : 3x_2 + x_3 + x_9 \geq 10$

$c_2 : -x_3 + 5x_4 \geq 4$

$c_3 : x_1 + x_5 + 2x_6 \geq 8$

$c_4 : x_7 - x_6 \geq 0$

$c_5 : 2x_8 - x_6 + x_9 \geq 0$

$c_6 : -x_3 - x_4 - x_7 - x_8 \geq -4$

$x_i, y \in \{-10, 10\}$

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$ @3

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle y \geq 2 \rangle$

$\langle x_5 \leq 2 \rangle$ @5

$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$c_5 \rightarrow \langle x_8 \geq 1 \rangle$

$c_6 \rightarrow \bot$

Constraint c_2
Propagated $x_4 \geq 1$
Decision level 3

Constraints

$$c_1 : 3x_2 + x_3 + x_9 \geq 10$$

$$x_3 + 5x_4 \geq 4$$

$$c_5 : 2x_8 + x_5 + 2x_6 \geq 8$$

$$-x_6 \geq 0$$

$$c_6 : -x_3 - x_4 - x_7 - x_8 \geq -4$$

$$x_i, y \in \{-10, 10\}$$

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$ @3

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle y \geq 2 \rangle$ @4

$\langle x_5 \leq 2 \rangle$ @5

$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$c_5 \rightarrow \langle x_8 \geq 1 \rangle$

$c_6 \rightarrow \bot$

Constraint c_6 Declares conflict

Constraints

$c_1 : 3x_2 + x_3 + x_9 \geq 10$

$c_2 : -x_3 + 5x_4 \geq 4$

$c_3 : x_1 + x_5 + 2x_6 \geq 8$

$c_4 : x_7 - x_6 \geq 0$

$c_5 : 2x_8 - x_6 + x_9 \geq 0$

$c_6 : -x_3 - x_4 - x_7 - x_8 \geq -4$

$x_i, y \in \{-10, 10\}$

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle y \geq 2 \rangle$ @4

$\langle x_5 \leq 2 \rangle$ @5

$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$c_5 \rightarrow \langle x_8 \geq 1 \rangle$

$c_6 \rightarrow \bot$

Classical Backtracking

Constraints

$$c_1 : 3x_2 + x_3 + x_9 \geq 10$$

$$c_2 : -x_3 + 5x_4 \geq 4$$

$$c_3 : x_1 + x_5 + 2x_6 \geq 8$$

$$c_4 : x_7 - x_6 \geq 0$$

$$c_5 : 2x_8 - x_6 + x_9 \geq 0$$

$$c_6 : -x_3 - x_4 - x_7 - x_8 \geq -4$$

$$x_i, y \in \{-10, 10\}$$

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle y \geq 2 \rangle$ @4

~~$\langle x_5 \leq 2 \rangle$ @5~~

$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$c_5 \rightarrow \langle x_8 \geq 1 \rangle$

$c_6 \rightarrow \bot$

Classical Backtracking

Undo decision

Constraints

$$c_1 : 3x_2 + x_3 + x_9 \geq 10$$

$$c_2 : -x_3 + 5x_4 \geq 4$$

$$c_3 : x_1 + x_5 + 2x_6 \geq 8$$

$$c_4 : x_7 - x_6 \geq 0$$

$$c_5 : 2x_8 - x_6 + x_9 \geq 0$$

$$c_6 : -x_3 - x_4 - x_7 - x_8 \geq -4$$

$$x_i, y \in \{-10, 10\}$$

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle y \geq 2 \rangle$ @4

$\langle x_5 \leq 2 \rangle$ @5

$\langle x_5 \geq 3 \rangle$

$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$c_5 \rightarrow \langle x_8 \geq 1 \rangle$

$c_6 \rightarrow \bot$

Classical Backtracking

Post negation and continue

Constraints

$$c_1 : 3x_2 + x_3 + x_9 \geq 10$$

$$c_2 : -x_3 + 5x_4 \geq 4$$

$$c_3 : x_1 + x_5 + 2x_6 \geq 8$$

$$c_4 : x_7 - x_6 \geq 0$$

$$c_5 : 2x_8 - x_6 + x_9 \geq 0$$

$$c_6 : -x_3 - x_4 - x_7 - x_8 \geq -4$$

$$x_i, y \in \{-10, 10\}$$

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$ @3

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle y \geq 2 \rangle$ @4

$\langle x_5 \leq 2 \rangle$ @5

$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

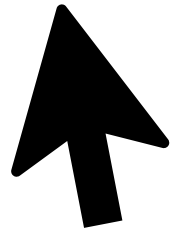
$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$c_5 \rightarrow \langle x_8 \geq 1 \rangle$

$c_6 \rightarrow \bot$

Conflict Analysis

$$\overset{@3}{\langle x_3 \geq 1 \rangle} \wedge \overset{@3}{\langle x_4 \geq 1 \rangle} \wedge \overset{@1}{\langle x_9 \leq 0 \rangle} \implies \langle x_6 \leq 1 \rangle$$



Derive nogood based on constraints

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$ @3

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle x_6 \leq 1 \rangle$

Conflict Analysis

$$\overset{@3}{\langle x_3 \geq 1 \rangle} \wedge \overset{@3}{\langle x_4 \geq 1 \rangle} \wedge \overset{@1}{\langle x_9 \leq 0 \rangle} \implies \langle x_6 \leq 1 \rangle$$

Backjump and propagate:
Decision level 3

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$ @3

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle x_6 \leq 1 \rangle$

Conflict Analysis

$$\overset{@3}{\langle x_3 \geq 1 \rangle} \wedge \overset{@3}{\langle x_4 \geq 1 \rangle} \wedge \overset{@1}{\langle x_9 \leq 0 \rangle} \implies \langle x_6 \leq 1 \rangle$$

Backjump and propagate:
Decision level 3

Skipped
Decision level 4!

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$ @3

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle y \geq 2 \rangle$ @4

$\langle x_5 \leq 2 \rangle$ @5

$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$c_5 \rightarrow \langle x_8 \geq 1 \rangle$

$c_6 \rightarrow \bot$

Constraint c_6 Declares conflict

Constraints

$c_1 : 3x_2 + x_3 + x_9 \geq 10$

$c_2 : -x_3 + 5x_4 \geq 4$

$c_3 : x_1 + x_5 + 2x_6 \geq 8$

$c_4 : x_7 - x_6 \geq 0$

$c_5 : 2x_8 - x_6 + x_9 \geq 0$

$c_6 : -x_3 - x_4 - x_7 - x_8 \geq -4$

$x_i, y \in \{-10, 10\}$

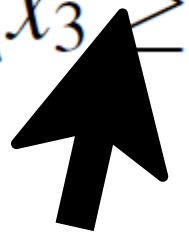
$$c_6: -x_3 - x_4 - x_7 - x_8 \geq -4$$

$$\langle x_3 \geq 1 \rangle^{@3} \wedge \langle x_4 \geq 1 \rangle^{@3} \wedge \langle x_7 \geq 2 \rangle^{@5} \wedge \langle x_8 \geq 1 \rangle^{@5} \implies \bot$$

Conflict Nogood

$$c_6: -x_3 - x_4 - x_7 - x_8 \geq -4$$

$$\langle x_3 \geq 1 \rangle^{@3} \wedge \langle x_4 \geq 1 \rangle^{@3} \wedge \langle x_7 \geq 2 \rangle^{@5} \wedge \langle x_8 \geq 1 \rangle^{@5} \implies \bot$$



Decision level of assignment

$$c_6 : -x_3 - x_4 - x_7 - x_8 \geq -4$$

$$\overset{@3}{\langle x_3 \geq 1 \rangle} \wedge \overset{@3}{\langle x_4 \geq 1 \rangle} \wedge \overset{@5}{\langle x_7 \geq 2 \rangle} \wedge \boxed{\overset{@5}{\langle x_8 \geq 1 \rangle}} \Longrightarrow \bot$$

Trail (Partial)

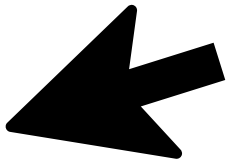
$\langle x_5 \leq 2 \rangle$ @5

$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$\boxed{c_5 \rightarrow \langle x_8 \geq 1 \rangle}$

$c_6 \rightarrow \bot$

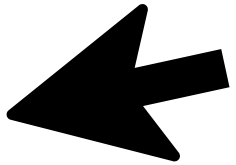


Last assigned atomic constraint

$$\boldsymbol{c_6: -x_3 - x_4 - x_7 - x_8 \geq -4}$$

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \boxed{\overset{@5}{\langle x_8 \geq 1\rangle}} \Longrightarrow \bot$$

$$c_5: 2x_8 - x_6 + x_9 \geq 0$$



$$\overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \Longrightarrow \boxed{\overset{@5}{\langle x_8 \geq 1\rangle}}$$

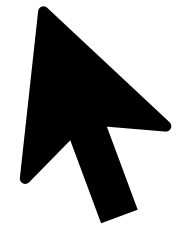
$$\boldsymbol{c_6: -x_3 - x_4 - x_7 - x_8 \geq -4}$$

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \boxed{\overset{@5}{\langle x_8 \geq 1\rangle}} \Longrightarrow \bot$$

Replace $\langle x_8 \geq 1\rangle$ with its reason

$$c_5: 2x_8 - x_6 + x_9 \geq 0$$

$$\overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \Longrightarrow \boxed{\overset{@5}{\langle x_8 \geq 1\rangle}}$$



$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \boxed{\overset{@5}{\langle x_8 \geq 1\rangle}} \implies \bot$$

$$\overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \boxed{\overset{@5}{\langle x_8 \geq 1\rangle}}$$

New nogood!

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \bot$$

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \bot$$

Trail (Partial)

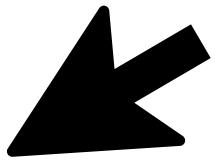
$\langle x_5 \leq 2\rangle$ @5

$c_3 \rightarrow \langle x_6 \geq 2\rangle$

$c_4 \rightarrow \langle x_7 \geq 2\rangle$

$c_5 \rightarrow \langle x_8 \geq 1\rangle$

$c_6 \rightarrow \bot$



Removed dependency on $\langle x_8 \geq 1\rangle$

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \bot$$

Trail (Partial)

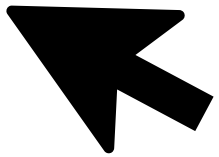
$\langle x_5 \leq 2\rangle$ @5

$c_3 \rightarrow \langle x_6 \geq 2\rangle$

$c_4 \rightarrow \langle x_7 \geq 2\rangle$

$c_5 \rightarrow \langle x_8 \geq 1\rangle$

$c_6 \rightarrow \bot$



Repeat procedure from $\langle x_7 \geq 2\rangle$

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \boxed{\overset{@5}{\langle x_7 \geq 2\rangle}} \wedge \overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \bot$$

$$\boldsymbol{c_4 : x_7 - x_6 \geq 0}$$

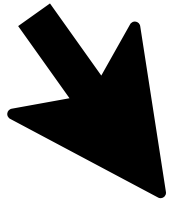
$$\overset{@5}{\langle x_6 \geq 2\rangle} \implies \boxed{\overset{@5}{\langle x_7 \geq 2\rangle}}$$

$$\langle x_3 \geq 1 \rangle^{@3} \wedge \langle x_4 \geq 1 \rangle^{@3} \wedge \langle x_7 \geq 2 \rangle^{@5} \wedge \langle x_6 \geq 2 \rangle^{@5} \wedge \langle x_9 \leq 0 \rangle^{@1} \implies \bot$$

$$\boldsymbol{c_4: x_7 - x_6 \geq 0}$$

New nogood!

$$\langle x_6 \geq 2 \rangle^{@5} \implies \langle x_7 \geq 2 \rangle^{@5}$$



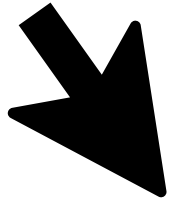
$$\langle x_3 \geq 1 \rangle^{@3} \wedge \langle x_4 \geq 1 \rangle^{@3} \wedge \langle x_6 \geq 2 \rangle^{@5} \wedge \langle x_9 \leq 0 \rangle^{@1} \implies \bot$$

$$\langle x_3 \geq 1 \rangle^{@3} \wedge \langle x_4 \geq 1 \rangle^{@3} \wedge \langle x_7 \geq 2 \rangle^{@5} \wedge \langle x_6 \geq 2 \rangle^{@5} \wedge \langle x_9 \leq 0 \rangle^{@1} \implies \bot$$

$$c_4 : x_7 - x_6 \geq 0$$

$$\langle x_6 \geq 2 \rangle^{@5} \implies \langle x_7 \geq 2 \rangle^{@5}$$

Propagates?



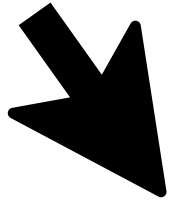
$$\langle x_3 \geq 1 \rangle^{@3} \wedge \langle x_4 \geq 1 \rangle^{@3} \wedge \langle x_6 \geq 2 \rangle^{@5} \wedge \langle x_9 \leq 0 \rangle^{@1} \implies \bot$$

$$\langle x_3 \geq 1 \rangle^{@3} \wedge \langle x_4 \geq 1 \rangle^{@3} \wedge \langle x_7 \geq 2 \rangle^{@5} \wedge \langle x_6 \geq 2 \rangle^{@5} \wedge \langle x_9 \leq 0 \rangle^{@1} \implies \bot$$

Propagates at decision level 3!

"asserting" $c_4 : x_7 - x_6 \geq 0$

$$\langle x_6 \geq 2 \rangle^{@5} \implies \langle x_7 \geq 2 \rangle^{@5}$$



$$\langle x_3 \geq 1 \rangle^{@3} \wedge \langle x_4 \geq 1 \rangle^{@3} \wedge \langle x_6 \geq 2 \rangle^{@5} \wedge \langle x_9 \leq 0 \rangle^{@1} \implies \bot$$

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \bot$$

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \overset{@3}{\langle x_6 \leq 1\rangle}$$

Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$ @3

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle y \geq 2 \rangle$ @4

$\langle x_5 \leq 2 \rangle$ @5

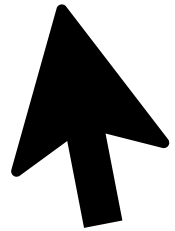
$c_3 \rightarrow \langle x_6 \geq 2 \rangle$

$c_4 \rightarrow \langle x_7 \geq 2 \rangle$

$c_5 \rightarrow \langle x_8 \geq 1 \rangle$

$c_6 \rightarrow \bot$

$$\overset{@3}{\langle x_3 \geq 1 \rangle} \wedge \overset{@3}{\langle x_4 \geq 1 \rangle} \wedge \overset{@1}{\langle x_9 \leq 0 \rangle} \implies \overset{@3}{\langle x_6 \leq 1 \rangle}$$



Trail

$\langle x_9 \leq 0 \rangle$ @1

$\langle x_1 \leq 2 \rangle$ @2

$\langle x_2 \leq 3 \rangle$ @3

$c_1 \rightarrow \langle x_3 \geq 1 \rangle$

$c_2 \rightarrow \langle x_4 \geq 1 \rangle$

$\langle x_6 \leq 1 \rangle$

$$\overset{@3}{\langle x_3 \geq 1 \rangle} \wedge \overset{@3}{\langle x_4 \geq 1 \rangle} \wedge \overset{@1}{\langle x_9 \leq 0 \rangle} \Longrightarrow \overset{@3}{\langle x_6 \leq 1 \rangle}$$

Backjump and propagate:
Decision level 3

Conflict Analysis

Algorithm 1 Conflict Analysis

Input: conflict explanation $(A_1 \wedge \cdots \wedge A_n \implies \bot)$

Output: asserting nogood

```
N ← conflict explanation
while N is not asserting do
    Let A be the most recently assigned atomic constraint in N
    Let R ⟹ A be the explanation of A
    Replace A in N by R
end while
return N
```

Conflict Analysis

Algorithm 1 Conflict Analysis

Input: conflict explanation $(A_1 \wedge \cdots \wedge A_n \implies \bot)$

Output: asserting nogood

$N \leftarrow$ conflict explanation

while N is not asserting **do**

  Let A be the most recently assigned atomic constraint in N

  Let $R \implies A$ be the explanation of A

  Replace A in N by R

end while

return N

Conflict Analysis

Algorithm 1 Conflict Analysis

Input: conflict explanation $(A_1 \wedge \cdots \wedge A_n \implies \bot)$

Output: asserting nogood

$N \leftarrow$ conflict explanation

while N is not asserting **do**

Let A be the most recently assigned atomic constraint in N

Let $R \implies A$ be the explanation of A

Replace A in N by R

end while

return N

Conflict Analysis

Algorithm 1 Conflict Analysis

Input: conflict explanation $(A_1 \wedge \cdots \wedge A_n \implies \bot)$

Output: asserting nogood

$N \leftarrow$ conflict explanation
while N is not asserting **do**
  Let A be the most recently assigned atomic constraint in N
  Let $R \implies A$ be the explanation of A
  Replace A in N by R
end while
return N

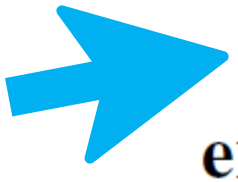
Conflict Analysis

Algorithm 1 Conflict Analysis

Input: conflict explanation $(A_1 \wedge \cdots \wedge A_n \implies \bot)$

Output: asserting nogood

$N \leftarrow$ conflict explanation
while N is not asserting **do**
  Let A be the most recently assigned atomic constraint in N
  Let $R \implies A$ be the explanation of A
  Replace A in N by R
end while
return N



Conflict Analysis

Algorithm 1 Conflict Analysis

Input: conflict explanation $(A_1 \wedge \cdots \wedge A_n \implies \bot)$

Output: asserting nogood

$N \leftarrow$ conflict explanation

while N is not asserting **do**

Let A be the most recently assigned atomic constraint in N

Let $R \implies A$ be the explanation of A

Replace A in N by R

end while

return N

Conflict Analysis

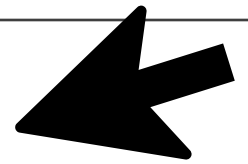
Algorithm 1 Conflict Analysis

Input: conflict explanation $(A_1 \wedge \cdots \wedge A_n \implies \bot)$

Output: asserting nogood

```
N ← conflict explanation
while N is not asserting do
    Let A be the most recently assigned atomic constraint in N
    Let R ⟹ A be the explanation of A
    Replace A in N by R
end while
return N
```

Why does this work?



Why does conflict analysis work?

Propagation-based rewriting

Conflict preservation

Soundness

Termination

Why does conflict analysis work?

Propagation-based rewriting

“we only rewrite propagated atomic constraints”

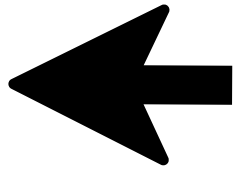
Why does conflict analysis work?

Propagation-based rewriting

“we only rewrite propagated atomic constraints”

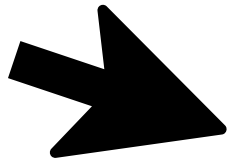
$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \overset{@5}{\langle x_8 \geq 1\rangle} \implies \bot$$

$$\overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \boxed{\overset{@5}{\langle x_8 \geq 1\rangle}}$$



$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \bot$$

Why does conflict analysis work?



Conflict preservation

“each new nogood is also conflicting”

Why does conflict analysis work?

Conflict preservation

“each new nogood is also conflicting”

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \overset{@5}{\langle x_8 \geq 1\rangle} \implies \bot$$

$$\overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \overset{@5}{\langle x_8 \geq 1\rangle}$$

Started with conflicting nogood

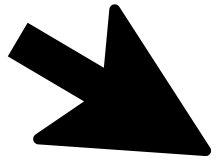
$$\overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \bot$$

Why does conflict analysis work?

Assigned “true (⊤)

Conflict preservation

“each new nogood is also conflicting”



$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \overset{@5}{\langle x_8 \geq 1\rangle} \Longrightarrow \bot$$

$$\overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \Longrightarrow \overset{@5}{\langle x_8 \geq 1\rangle}$$

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \Longrightarrow \bot$$

Why does conflict analysis work?

Conflict preservation

“each new nogood is also conflicting”

$$\langle x_3 \geq 1\rangle^{@3} \wedge \langle x_4 \geq 1\rangle^{@3} \wedge \langle x_7 \geq 2\rangle^{@5} \wedge \langle x_8 \geq 1\rangle^{@5} \implies \bot$$

$$\langle x_6 \geq 2\rangle^{@5} \wedge \langle x_9 \leq 0\rangle^{@1} \implies \langle x_8 \geq 1\rangle^{@5}$$

Must be “true” (⊤) because propagating

$$\langle x_4 \geq 1\rangle^{@3} \wedge \langle x_7 \geq 2\rangle^{@5} \wedge \langle x_6 \geq 2\rangle^{@5} \wedge \langle x_9 \leq 0\rangle^{@1} \implies \bot$$

Why does conflict analysis work?

Conflict preservation

"each new nogood is also conflicting"

These must also "true" (⊤)

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \overset{@5}{\langle x_8 \geq 1\rangle} \implies \bot$$

$$\overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \overset{@5}{\langle x_8 \geq 1\rangle}$$

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@5}{\langle x_7 \geq 2\rangle} \wedge \overset{@5}{\langle x_6 \geq 2\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \bot$$

Why does conflict analysis work?

Soundness

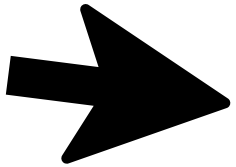
“new nogoods are implied by the problem”

Why does conflict analysis work?

Soundness

“new nogoods are implied by the problem”

Rewriting is sound



Explanations are sound

Why does conflict analysis work?

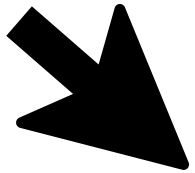
Termination

“eventually we derive an asserting nogood”

Why does conflict analysis work?

Termination

“eventually we derive an asserting nogood”



Atomic constraints are replaced based on recency

Finite trail size

Conflict Analysis

Algorithm 1 Conflict Analysis

Input: conflict explanation $(A_1 \wedge \cdots \wedge A_n \implies \bot)$
Output: asserting nogood

$N \leftarrow$ conflict explanation
while N is not asserting **do**
  Let A be the most recently assigned atomic constraint in N
  Let $R \implies A$ be the explanation of A
  Replace A in N by R
end while
return N =0

Propagation-based rewriting

Conflict preservation

Soundness

Termination

So far…

Conflict Analysis

Learn nogoods
by combining explanations

Potentially backjump

Up next…

Checker for learned nogoods

Conflict-Driven Constraint Learning

(Extended) Nogood propagation

More general learning

Extended nogoods

Hypercube linears

Checker

“How to verify the learned nogood?”

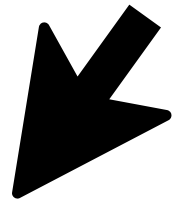
Checker

“How to verify the learned nogood?”

1. Assume the atomic constraints from nogood to true (⊤)

2. Derive conflict by following implication chain

Learned nogood we wish to verify



$$\langle x_3 \geq 1 \rangle \wedge \langle x_4 \geq 1 \rangle \wedge \langle x_6 \geq 2 \rangle \wedge \langle x_9 \leq 0 \rangle \implies \bot$$

$$\langle x_3 \geq 1 \rangle \wedge \langle x_4 \geq 1 \rangle \wedge \langle x_6 \geq 2 \rangle \wedge \langle x_9 \leq 0 \rangle \implies \bot$$

$$x_3 \in [1, \infty],\ x_4 \in [1, \infty],\ x_6 \in [2, \infty],\ x_9 \in [-\infty, 0]$$

$$x_3 \in [1, \infty], \; x_4 \in [1, \infty], \; x_6 \in [2, \infty], \; x_9 \in [-\infty, 0]$$

$$x_3 \in [1,\infty],\ x_4 \in [1,\infty],\ x_6 \in [2,\infty],\ x_9 \in [-\infty,0]$$

$$\langle x_6 \geq 2 \rangle \implies \langle x_7 \geq 2 \rangle$$

$$x_3 \in [1,\infty],\ x_4 \in [1,\infty],\ x_6 \in [2,\infty],\ x_9 \in [-\infty,0]$$

$$\langle x_6 \geq 2 \rangle \implies \langle x_7 \geq 2 \rangle$$

$$x_3 \in [1,\infty],\ x_4 \in [1,\infty],\ x_6 \in [2,\infty],\ x_9 \in [-\infty,0],\ x_7 \in [2,\infty]$$

$$x_3 \in [1,\infty],\ x_4 \in [1,\infty],\ x_6 \in [2,\infty],\ x_9 \in [-\infty,0],\ x_7 \in [2,\infty]$$

$$\langle x_6 \geq 2 \rangle \wedge \langle x_9 \leq 0 \rangle \implies \langle x_8 \geq 1 \rangle$$

$$x_3 \in [1,\infty],\ x_4 \in [1,\infty],\ x_6 \in [2,\infty],\ x_9 \in [-\infty,0],\ x_7 \in [2,\infty]$$

$$\langle x_6 \geq 2 \rangle \wedge \langle x_9 \leq 0 \rangle \implies \langle x_8 \geq 1 \rangle$$

$$x_3 \in [1,\infty],\ x_4 \in [1,\infty],\ x_6 \in [2,\infty],\ x_9 \in [-\infty,0],\ x_7 \in [2,\infty],\ x_8 \in [1,\infty]$$

$$x_3 \in [1,\infty],\ x_4 \in [1,\infty],\ x_6 \in [2,\infty],\ x_9 \in [-\infty,0],\ x_7 \in [2,\infty],\ x_8 \in [1,\infty]$$

$$\langle x_3 \geq 1 \rangle \wedge \langle x_4 \geq 1 \rangle \wedge \langle x_7 \geq 2 \rangle \wedge \langle x_8 \geq 1 \rangle \implies \bot$$

$$x_3 \in [1,\infty],\ x_4 \in [1,\infty],\ x_6 \in [2,\infty],\ x_9 \in [-\infty,0],\ x_7 \in [2,\infty],\ x_8 \in [1,\infty]$$

$$\langle x_3 \geq 1 \rangle \wedge \langle x_4 \geq 1 \rangle \wedge \langle x_7 \geq 2 \rangle \wedge \langle x_8 \geq 1 \rangle \implies \bot$$

Conflict

Initial nogood is correct

(Assuming explanations are sound)

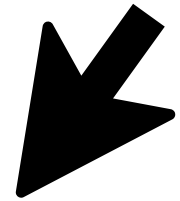
Search

Make a decision

Propagate based on constraints

If conflict, backtrack

Constraint Conflict-Driven ~~Clause~~ Learning (CDCL)



Make a decision

Conflict-driven branching (VSIDS)

Propagate based on constraints

"atomic constraints appearing in conflicts are important"

If conflict, backtrack

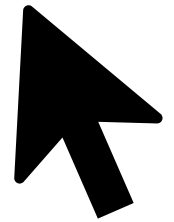
Constraint Conflict-Driven ~~Clause~~ Learning (CDCL)

Make a decision

Conflict-driven branching (VSIDS)

Propagate based on constraints …and learned nogoods

If conflict, backtrack

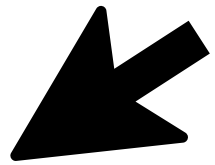


Constraint Conflict-Driven ~~Clause~~ Learning (CDCL)

Make a decision Conflict-driven branching (VSIDS)

Propagate based on constraints …and learned nogoods

If conflict, ~~backtrack~~



Conflict analysis

Add learned nogood to database

Backjump

Constraint Conflict-Driven ~~Clause~~ Learning (CDCL)

Make a decision Conflict-driven branching (VSIDS)

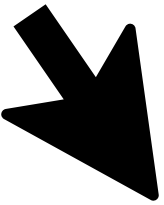
Propagate based on constraints …and learned nogoods

If conflict, ~~backtrack~~

Conflict analysis

Add learned nogood to database

Backjump



Restarts

Nogood Management

Constraint Conflict-Driven ~~Clause~~ Learning (CDCL)

Make a decision

Conflict-driven branching (VSIDS)

Propagate based on constraints …and learned nogoods

If conflict, ~~backtrack~~

Conflict analysis

Add learned nogood to database

Backjump

Restarts

Nogood Management

Nogood Propagation

$$\langle x_3 \geq 1 \rangle^{@3} \wedge \langle x_4 \geq 1 \rangle^{@3} \wedge \langle x_9 \leq 0 \rangle^{@1} \implies \langle x_6 \leq 1 \rangle$$

Atomic constraint

$\{?, \top, \bot\}$

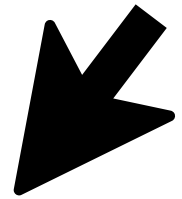
Nogood Propagation

$$\overset{@3}{\langle x_3 \geq 1\rangle} \wedge \overset{@3}{\langle x_4 \geq 1\rangle} \wedge \overset{@1}{\langle x_9 \leq 0\rangle} \implies \langle x_6 \leq 1\rangle$$

Atomic constraint

$\{?, \top, \bot\}$

True



$x_3 \in [6, 10]$ $\langle x_3 \geq 5\rangle$ $\top$

Nogood Propagation

$$\langle x_3 \geq 1 \rangle^{@3} \wedge \langle x_4 \geq 1 \rangle^{@3} \wedge \langle x_9 \leq 0 \rangle^{@1} \implies \langle x_6 \leq 1 \rangle$$

Atomic constraint

$\{?, \top, \bot\}$

$x_3 \in [6, 10]$ $\quad \langle x_3 \geq 5 \rangle$ $\quad \top$

$x_3 \in [6, 10]$ $\quad \langle x_3 \leq 5 \rangle$ $\quad \bot$

False

Nogood Propagation

$$\langle x_3 \geq 1 \rangle^{@3} \wedge \langle x_4 \geq 1 \rangle^{@3} \wedge \langle x_9 \leq 0 \rangle^{@1} \implies \langle x_6 \leq 1 \rangle$$

Atomic constraint

$\{?, \top, \bot\}$

$x_3 \in [6, 10]$ $\quad \langle x_3 \geq 5 \rangle$ $\quad \top$

$x_3 \in [6, 10]$ $\quad \langle x_3 \leq 5 \rangle$ $\quad \bot$

$\mathbf{x_3 \in [0, 10]}$ $\quad \mathbf{\langle x_3 \geq 5 \rangle}$ $\quad ?$

Unassigned

Nogood Propagation

$$\boldsymbol{A} \wedge \boldsymbol{B} \wedge \boldsymbol{C} \wedge \boldsymbol{D} \Rightarrow \bot$$

$$\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D} \in \{?, \top, \bot\}$$

No propagation when all unassigned

$$\boldsymbol{A} \wedge \boldsymbol{B} \wedge \boldsymbol{C} \wedge \boldsymbol{D} \Rightarrow \bot$$

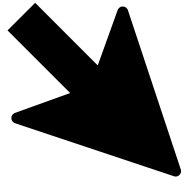
$$\boldsymbol{A} = ?$$

$$\boldsymbol{B} = ?$$

$$\boldsymbol{C} = ?$$

$$\boldsymbol{D} = ?$$

No propagation…



$$A \wedge B \wedge C \wedge D \Rightarrow \bot$$

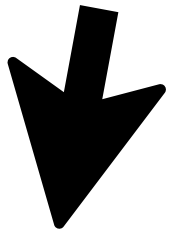
$$A = \top$$

$$B = ?$$

$$C = ?$$

$$D = ?$$

Still no propagation…



$$\boldsymbol{A} \wedge \boldsymbol{B} \wedge \boldsymbol{C} \wedge \boldsymbol{D} \Rightarrow \bot$$

$$\boldsymbol{A} = \top$$

$$\boldsymbol{B} = \top$$

$$\boldsymbol{C} = ?$$

$$\boldsymbol{D} = ?$$

Propagation happens!

$$\boldsymbol{A} \wedge \boldsymbol{B} \wedge \boldsymbol{C} \wedge \boldsymbol{D} \Rightarrow \bot$$

$$\boldsymbol{A} = \top$$

$$\boldsymbol{B} = \top$$

$$\boldsymbol{C} = \top$$

$$\boldsymbol{D} = ?$$

D forced to $\bot$

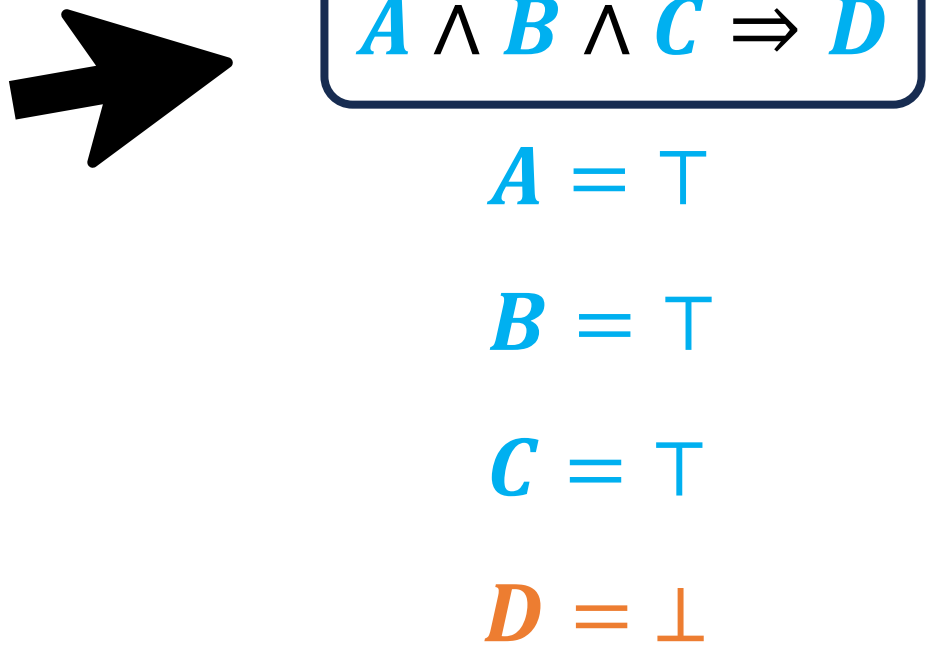
$$A \wedge B \wedge C \wedge D \Rightarrow \bot$$

$$A = \top$$

$$B = \top$$

$$C = \top$$

$$D = \bot$$

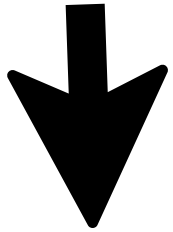

$A \wedge B \wedge C \wedge D \Rightarrow \bot$
$A \wedge B \wedge C \Rightarrow \overline{D}$
$A = \top$
$B = \top$
$C = \top$
$D = \bot$

Nogood Propagation

$$A_1 \wedge A_2 \wedge \cdots \wedge A_n \Rightarrow \bot$$

Nogood Propagation

$$A_1 \wedge A_2 \wedge \cdots \wedge A_n \Rightarrow \bot$$



Conflict

$A_i = \top$

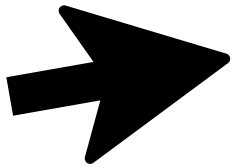
Nogood Propagation

$$A_1 \wedge A_2 \wedge \cdots \wedge A_n \Rightarrow \bot$$

Conflict

$$A_i = \top$$

"unit propagation"



Propagation

$$A_{1\ldots(n-1)} = \top$$

$$A_n = \bot$$

$$A_1 \wedge A_2 \wedge \cdots \wedge A_{n-1} \Rightarrow \overline{A_n}$$

Nogood Propagation

$$A_1 \wedge A_2 \wedge \cdots \wedge A_n \Rightarrow \bot$$

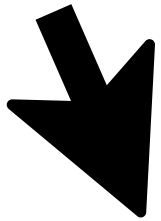
Conflict

$$A_i = \top$$

Propagation

$$A_{1\ldots(n-1)} = \top$$

$$A_n = \bot$$



Efficient propagation algorithm
"two-watcher scheme"

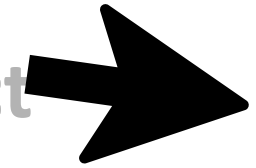
$$A_1 \wedge A_2 \wedge \cdots \wedge A_{n-1} \Rightarrow \overline{A_n}$$

Nogood Propagation

$$A_1 \wedge A_2 \wedge \cdots \wedge A_n \Rightarrow \bot$$

Conflict

$A_i = \top$



Can we do more propagation?

Propagation

$A_{1\ldots(n-1)} = \top$

$A_n = \bot$

Efficient propagation algorithm
"two-watcher scheme"

$$A_1 \wedge A_2 \wedge \cdots \wedge A_{n-1} \Rightarrow \overline{A_n}$$

Extended Propagation

$$\boldsymbol{A} \wedge \boldsymbol{B} \wedge \boldsymbol{C} \wedge \boldsymbol{D} \Rightarrow \bot$$

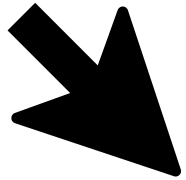
$$\boldsymbol{A} = ?$$

$$\boldsymbol{B} = ?$$

$$\boldsymbol{C} = ?$$

$$\boldsymbol{D} = ?$$

No propagation…



$$A \wedge B \wedge C \wedge D \Rightarrow \bot$$

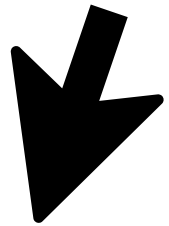
$$A = \top$$

$$B = ?$$

$$C = ?$$

$$D = ?$$

Still no propagation according to unit propagation…



$$\boldsymbol{A} \wedge \boldsymbol{B} \wedge \boldsymbol{C} \wedge \boldsymbol{D} \Rightarrow \bot$$

$$\boldsymbol{A} = \top$$

$$\boldsymbol{B} = \top$$

$$\boldsymbol{C} = ?$$

$$\boldsymbol{D} = ?$$

…but if we look at the atomic constraints…

$$\langle x \geq 5 \rangle \wedge \langle y \geq 2 \rangle \wedge \langle z \geq 1 \rangle \wedge \langle z \leq 3 \rangle \Rightarrow \bot$$

$$\langle x \geq 5 \rangle = \top$$

$$\langle y \geq 3 \rangle = \top$$

$$\langle z \geq 1 \rangle = ?$$

$$\langle z \leq 3 \rangle = ?$$

$$\langle x \geq 5 \rangle \wedge \langle y \geq 2 \rangle \wedge \langle z \geq 1 \rangle \wedge \langle z \leq 3 \rangle \Rightarrow \bot$$

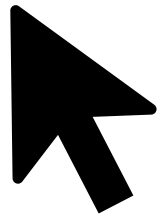
$$\langle x \geq 5 \rangle \wedge \langle y \geq 2 \rangle \Rightarrow \langle z \neq 1 \rangle \wedge \langle z \neq 2 \rangle \wedge \langle z \neq 3 \rangle$$

$\langle x \geq 5 \rangle = \top$

$\langle y \geq 3 \rangle = \top$

$\langle z \geq 1 \rangle = ?$

$\langle z \leq 3 \rangle = ?$



We can propagate!

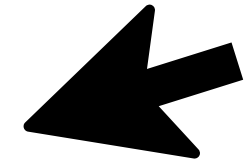
“extended nogood propagation”

Conflict Analysis with Extended Nogood Propagation

Algorithm 1 Conflict Analysis

Input: conflict explanation $(A_1 \wedge \cdots \wedge A_n \implies \bot)$

Output: ~~asserting nogood~~ propagating nogood



$N \leftarrow$ conflict explanation

while ~~N is not asserting~~ **do** N is not propagating

Let A be the most recently assigned atomic constraint in N

Let $R \implies A$ be the explanation of A

Replace A in N by R

end while

return N

Conflict Analysis with Extended Nogood Propagation

Algorithm 1 Conflict Analysis

Input: conflict explanation $(A_1 \wedge \cdots \wedge A_n \implies \bot)$

Output: ~~asserting nogood~~ propagating nogood

$N \leftarrow$ conflict explanation

while ~~N is not asserting~~ **do** N is not propagating

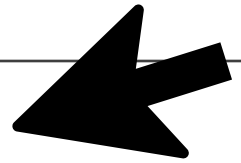
Let A be the most recently assigned atomic constraint in N

Let $R \implies A$ be the explanation of A

Replace A in N by R

end while

return N



More general learning!

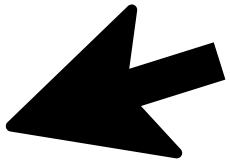
Conflict analysis
is about
combining constraints!

Problem with two constraints

$$\sum \boldsymbol{w_i x_i \geq k} \quad \wedge \quad \sum \boldsymbol{w_i x_i \leq k - 1}$$

Problem with two constraints

$$\sum w_i x_i \geq k \quad \wedge \quad \sum w_i x_i \leq k - 1$$



Infeasible!

Easy problem, but exponential time for our search with conflict analysis!

Cutting planes: a better way…

Multiply by -1

$$\sum w_i x_i \geq k \longrightarrow \sum -w_i x_i \leq -k$$

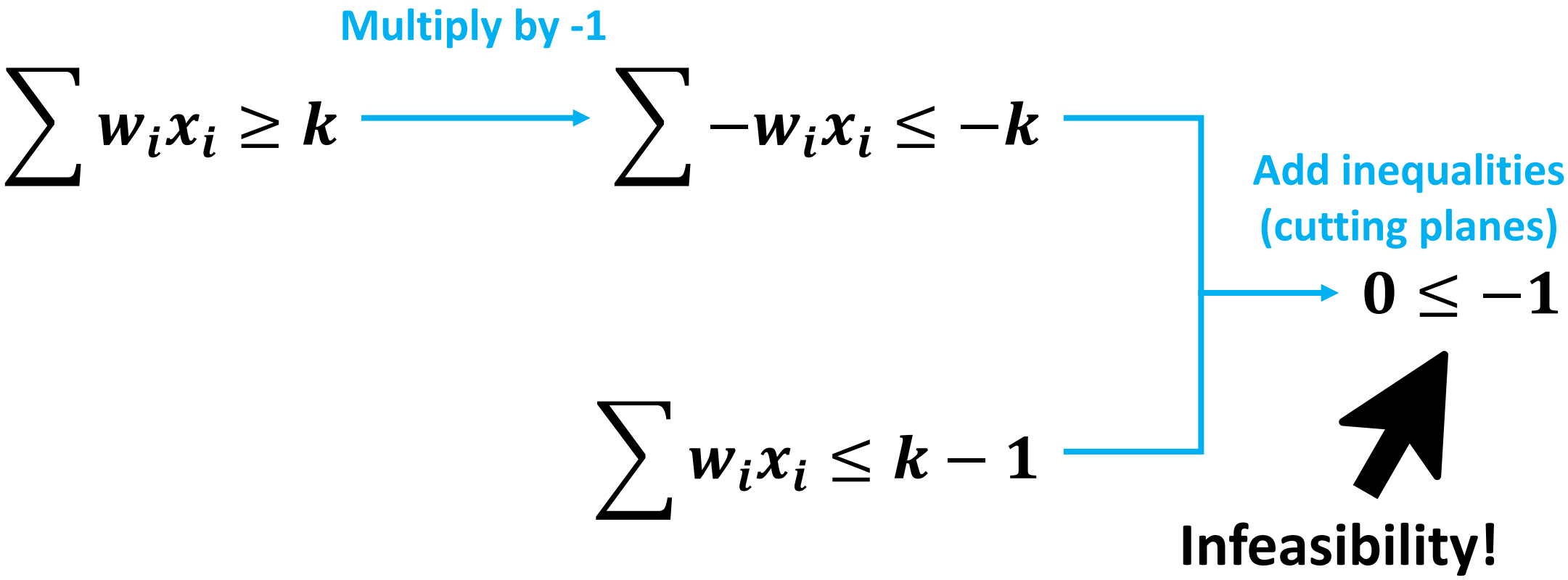
$$\sum w_i x_i \leq k - 1$$

Multiply by -1

$$\sum w_i x_i \geq k \longrightarrow \sum -w_i x_i \leq -k$$

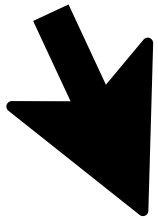
Add inequalities
(cutting planes)

$$\sum w_i x_i \leq k - 1$$



Exponentially better
(on this example)

Combining Constraints



Complete

1. “Rewrite” nogoods

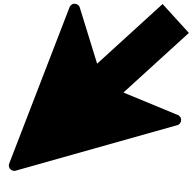
two nogoods → new nogood

Combining Constraints

1. “Rewrite” nogoods

two nogoods → new nogood

Complete



2. Cutting planes

two inequalities→ new inequality

Incomplete

(complete only in real-valued space)

Combining Constraints

1. “Rewrite” nogoods

two nogoods → new nogood

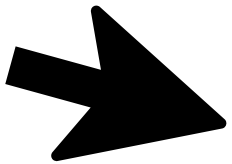
Complete

2. Cutting planes

two inequalities→ new inequality

Incomplete

(complete only in real-valued space)



Can we combine the two?

Hypercube Linear Constraints

$$A_1 \wedge A_2 \wedge \cdots \wedge A_n \Rightarrow \sum w_i x_i \geq k$$

Hypercube Linear Constraints

$$A_1 \wedge A_2 \wedge \cdots \wedge A_n \Rightarrow \sum w_i x_i \geq k$$

Generalises nogoods

$$A_1 \wedge A_2 \wedge \cdots \wedge A_n \Rightarrow \bot$$

Generalises linear inequalities

$$\top \Rightarrow \sum w_i x_i \geq k$$

Conflict Analysis with Hypercube Linears

Algorithm 1 Conflict Analysis

Conflict hypercube linear

Input: ~~conflict explanation $(A_1 \wedge \cdots \wedge A_n \Longrightarrow \bot)$~~

Output: ~~asserting nogood~~ propagating hypercube linear

$N \leftarrow$ conflict explanation

while ~~N is not asserting~~ **do** N is not propagating

Let A be the most recently assigned atomic constraint in N

~~Let $R \Longrightarrow A$ be the explanation of A~~ Hypercube linear HL that propagated A

~~Replace A in N by R~~

end while

N ← Combine HL and N

return N

Conflict Analysis with Hypercube Linears

Algorithm 1 Conflict Analysis

Input: ~~conflict explanation $(A_1 \wedge \cdots \wedge A_n \implies \bot)$~~ Conflict hypercube linear

Output: ~~asserting nogood~~ propagating hypercube linear

$N \leftarrow$ conflict explanation

while ~~N is not asserting~~ **do** N is not propagating

Let A be the most recently assigned atomic constraint in N

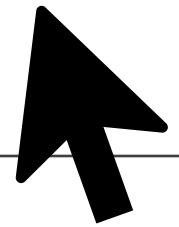
~~Let $R \implies A$ be the explanation of A~~ Hypercube linear HL that propagated A

~~Replace A in N by R~~

end while

N ← Combine HL and N

return N



Analogous as before!

Conflict Analysis with Hypercube Linears

Algorithm 1 Conflict Analysis

Conflict hypercube linear

Input: ~~conflict explanation $(A_1 \wedge \cdots \wedge A_n \implies \bot)$~~

Output: ~~asserting nogood~~ propagating hypercube linear

$N \leftarrow$ conflict explanation

while ~~N is not asserting~~ **do** N is not propagating

Let A be the most recently assigned atomic constraint in N

~~Let $R \implies A$ be the explanation of A~~ Hypercube linear HL that propagated A

~~Replace A in N by R~~

end while

return N

N ← Combine HL and N

More complex!
(see our CPAIOR'26 paper…)

Summary

Conflict Analysis

Combine constraints

Nogoods

Extended nogoods

Hypercube Linears

Conflict-Driven Constraint Learning

1. Learning new constraints
2. Influence branching
3. Nogood management
4. Restarts

Checker for learned nogoods

Next time…

Certificates and Proof Systems

Combine everything we have done so far
to produce independently verifiable certificates
about infeasibility/optimality

Discussion on recent approaches